Production and Capacity Planning

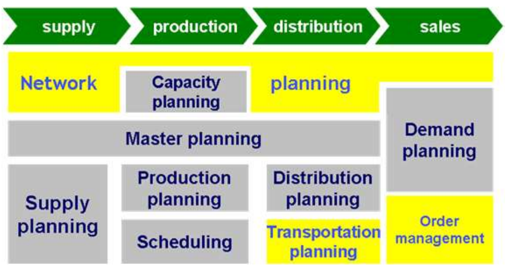
2019
Dr. József Váncza

Lecturer: Dr. Peter Zentay

Gyártástudomány és -technológia Tanszék
Department of Manufacturing Science and Engineering
Budapesti Műszaki és Gazdaságtudományi Egyetem
Budapest University of Technology and Economics

Problem statement

- Matching demands and capacities
  - In the future
  - On a longer horizon, for each time period
- At several levels of aggregation
  - Problem size
  - Uncertainty
- Complexity
  - How much time do we need to get response?
- Material flow vs. capacities
  - Traditionally decomposed
- Result
  - Plan for the future
    - Production, supply, distribution, sales …
  - Capacity plan
    - On the availability of machine and human resources



Principles

- Planning capacities and load together
  - Limited time-variant capacities
  - No fixed lead time
- Aggregation
  - Due to complexity (and common sense)
  - Dimensions
    - Time
    - Product structure: no BOMs
    - Technology: no technological plans or routings
- Relaxation
  - Deliberately neglecting some constraints
- Optimisation
  - Looking for the best solution that satisfies constraints
  - Solution time vs. performance
- Decision support
  - Providing alternative solutions
  - "What-if" analysis

Method

- Basic model and its extensions
  - At relatively high aggregation level
  - Through, with less and less relaxations
  - Without decomposition
- Declarative modelling
  - Problem description - questioning - answer generation / separates /
  - Model + solution generation
- Suggested tool: Xpress-Mosel
  - FICO Xpress Optimization Suite Rel. 7.5 (MS Windows)
    - Program development environment
    - Modelling language
    - Solver

  http://www.fico.com/en/Products/DMTools/Pages/FICO-Xpress-Optimization-Suite.aspx
  - Free student version

  https://community.fico.com/download.jspa

Basic concepts

Production environment
- Resource capacities
  - Machines and workforce
- Products
  - Without structure (BOM)
  - Material demand
- Technology
  - Resource demand
- Inventory
- Business policies
  - All demand must be satisfied
  - Delay is possible/impossible
  - Capacity is fixed/extendible

•External environment
- Market demand
  - Between minimal and maximal limits
- External resources
- Profit
  - Prices
  - Costs
    - Production, inventory, delay, backorder, overtime …

Aggregated production and capacity planning

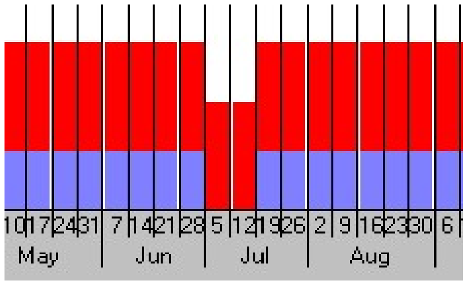
•Basic problem: given are
- Maximal demand for each product
- Minimal delivery for each product
- Net profit per product (price minus production cost)
- Resource demand of each product
- Resource capacity limits, for each periods
- Unit cost of inventory, per product and period
- Initial inventory of each product

•Assumptions
- Unsatisfied demand will be lost
- Material availability is unlimited

•Question
- How to maximize total profit by producing and selling products?
- Specifically: in each period and for each product
  - How much to produce?
  - How much to sell?
  - How much to stock?

Aggregated production and capacity planning (2)

- Modelling time
  - Discrete time periods (*time bucket*)
    - All important quantities will change in time
    - The length of time periods does not really matter, could be different
- Time-varying parameters
  - E.g., capacity limits
    - Internal
    - External
- Basic model
  - Later its extensions
- Formulation
  - Linear program (LP)
- Executable program
  - aggregate_planning



Aggregated production and capacity planning (3)

Basic model

Indices	
Product index	$i = 1,\ldots,m$
Resource index	$j = 1,\ldots,n$
Time periods' index (limited horizon)	$t = 1,\ldots,T$
Parameters	
Maximal demand, for each product	$\overline{d}_{it}$
Minimal delivery, for each product	$\underline{d}_{it}$
Resource demand	a_{ij}
Capacity limit	c_{jt}
Profit per product	p_i
Holding cost, per time period and product	h_i
Initial inventory, for each product	I_i^0

Aggregated production and capacity planning (4)

Basic model (ctd.)

Decision variables	
Production quantities, for each product and period	X_{it}
Sales quantities, for each product and period	S_{it}
Inventory levels, for each product and period	I_{it}

Optimization criterion (objective)

- Profit maximisation
  - Profit for sold quantities minus cost of inventory holding
  - No setup costs → no need to plan lots

$$\max \sum_{t=1}^{T} \sum_{i=1}^{m} p_i S_{it} - h_i I_{it}$$

Aggregated production and capacity planning (5)

•Constraints

- Amount to sell between maximal and minimal quantities

$$\underline{d}_{it} \leq S_{it} \leq \overline{d}_{it} \quad \forall i,t$$

- Resource capacity limits

$$\sum_{i=1}^{m} a_{ij} X_{it} \leq c_{jt} \quad \forall j,t$$

- Initial inventories

$$I_{i0} = I_i^0 \quad \forall i$$

- Balance equation (inventory holding – production – sales)
  - Connection between two subsequent time-periods

$$I_{it} = I_{it-1} + X_{it} - S_{it} \quad \forall i,t$$

- Integrity constraints

$$X_{it}, S_{it}, I_{it} \geq 0$$

Formulation: Linear program (LP)

- Problem class
  - Real-valued variables, linear objective function, linear constraints
- Generic problem formulation
  - Decision variables
  - Constraints
  - Objective (goal function)

$$\min\ cx, \qquad Ax \geq b, \qquad \forall i\ \ x_i \quad \text{constrained}$$

- Solution
  - Admissible solutions: polyhedron bounded by hyperplanes (in n dimensions)
  - Optimal solution: one vertex
- Methods
  - Simplex (Dantzig, 1940-50)
  - Internal point (Karmarkar, 1984)
  - Applicable even in very large scales ($n \approx 10^5$)
  - Successful applications

Linear program: simple example

- Problem statement: Simplified version of the previous one
  - How much to produce from given products, if we know
  - Maximal demand per products,
  - Profit per products,
  - Resource requirements of products
  - Capacity limits of resources
- No time dimension
  - One-period model
- Data

Product	i	1	2
Profit	p_i	45	60
Maximal demand	$\bar{d}_i$	100	50
Res. reg. of machine A	a_{iA}	15	10
Res. reg. of machine B	a_{iB}	15	35
Res. reg. of machine C	a_{iC}	15	5
Res. reg. of machine D	a_{iD}	25	14

Linear program: simple example (2)

Data (ctd.)

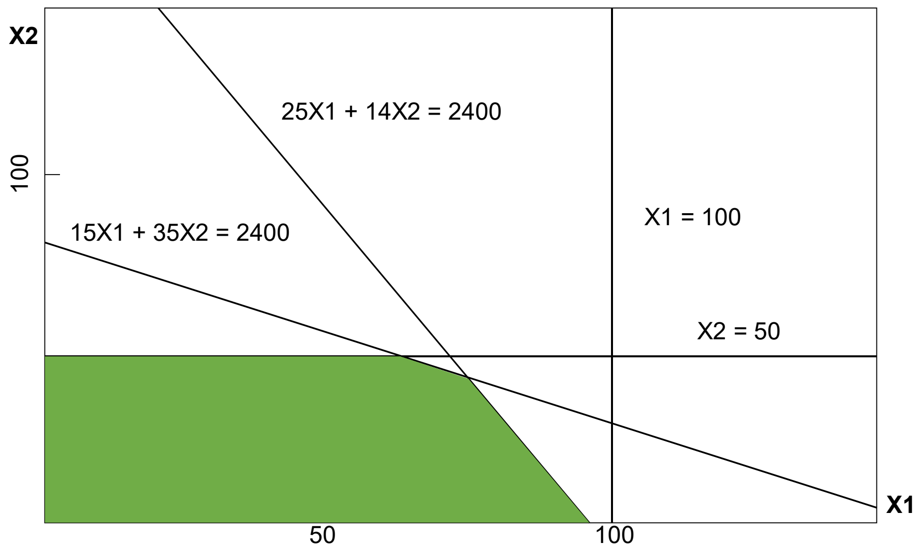
- Resources (machines)

Machine	j	A	B	C	D
Capacity	c_j	2400	2400	2400	2400

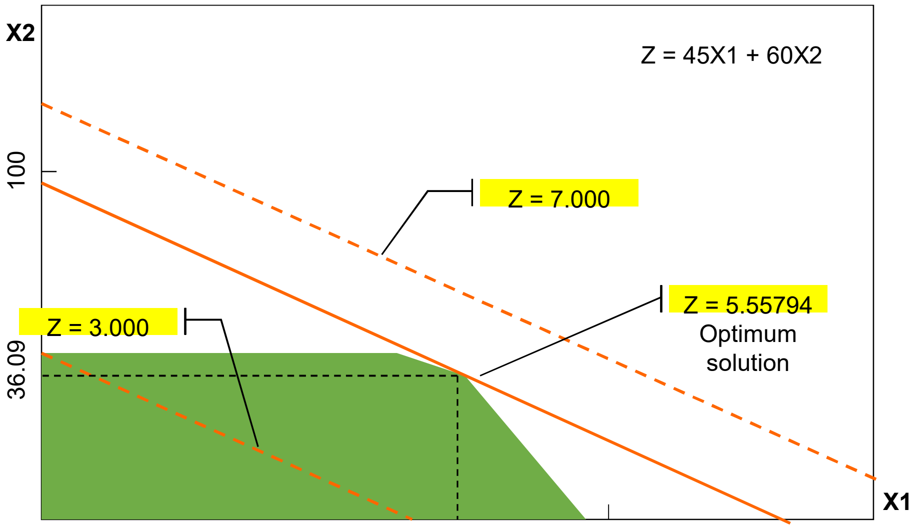
Program

- aggregate_planning_simple

Solution of the linear program



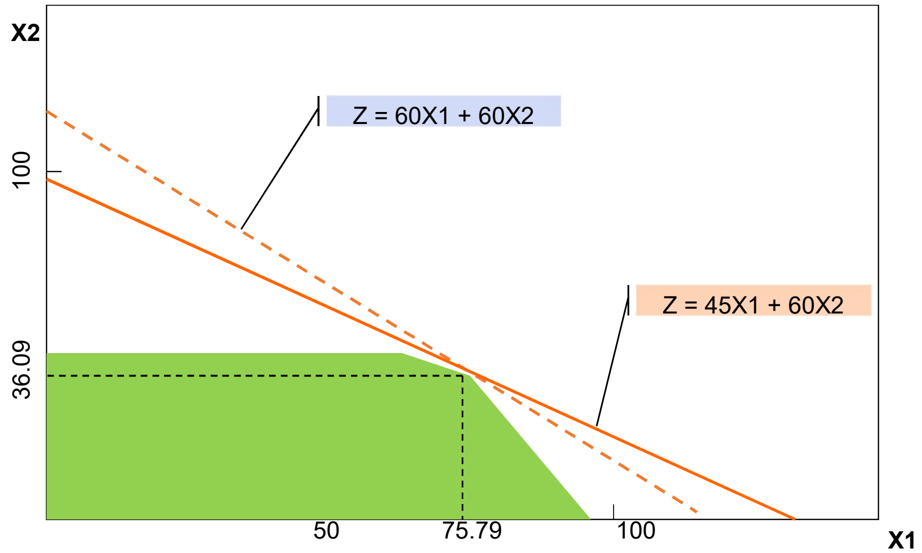
Solution of the linear program (2)



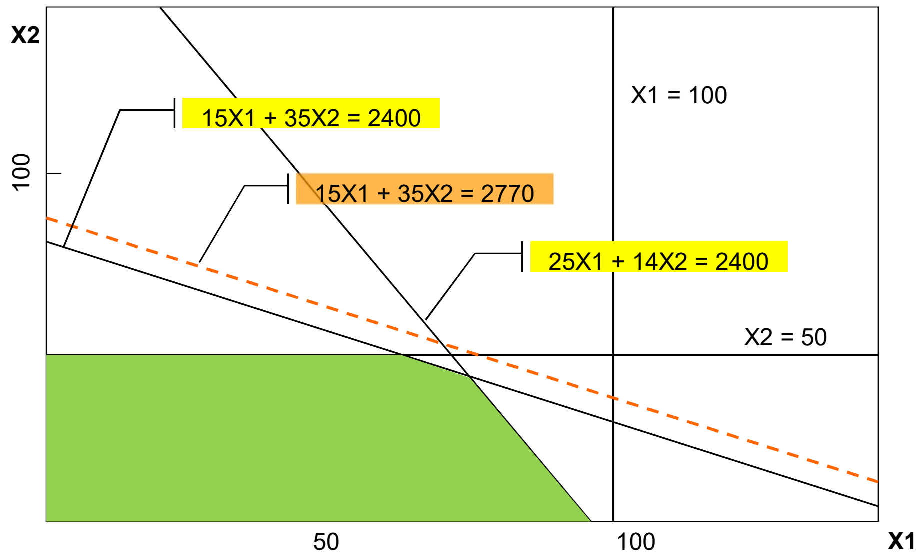
Sensitivity analysis

- If the input parameters change …
  - How does the result change?
- Changeable parameters
  - Parameters in the objective function
    - In our case, the profit per products
  - Parameters in the constraints
    - In our case, maximal demand and the resource requirements of products
  - „Right-hand side” parameters
    - In our case, the machine capacities
- How to modify?
  - So-called slack constraints
    - No effect on optimum, E.g., X2 <= 50
    - Easy to calculate when they begin to affect the optimum
  - Strong constraints
    - Determine the optimum

Sensitivity analysis (2)



Sensitivity analysis (3)



Original planning problem: example

Products

- Maximal and minimal demand, profit, holding cost
- Initial inventory

period	1	2	3	4	5	6	7	8
product	**max demand**							
P1	20	50	50	50	50	50	50	50
P2	30	30	40	20	30	40	40	40
	min sales							
P1	0	0	0	0	0	0	0	0
P2	10	10	10	10	0	0	0	0
	profit		**holding cost**		**init hold**			
P1	1500		20		0			
P2	1000		10		0			

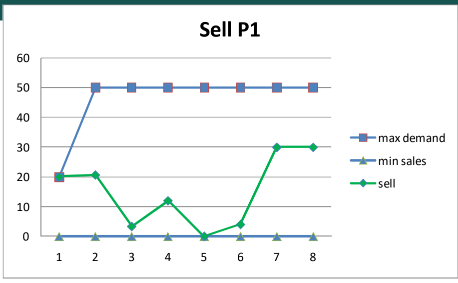
Original planning problem: example (2)

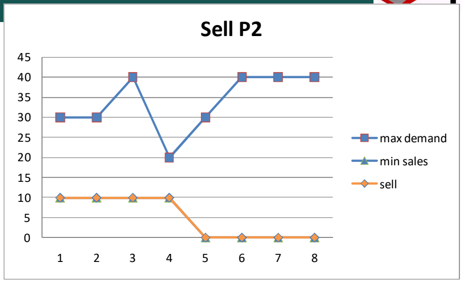
Resources

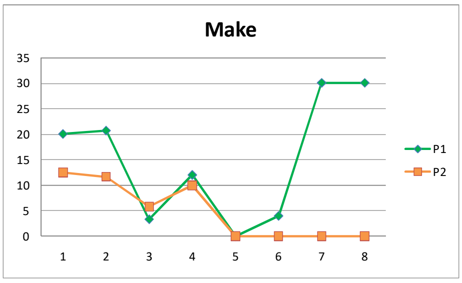
- capacities, resource requirements

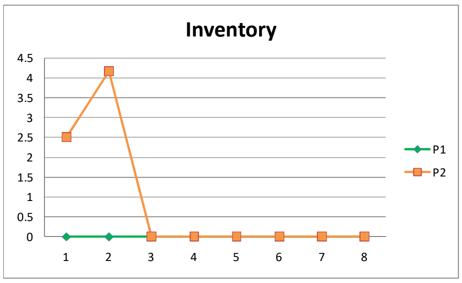
period	1	2	3	4	5	6	7	8
resource	**capacity**							
WS_A	250	250	60	100	0	20	200	240
WS_B	150	220	60	100	0	120	200	240
WS_C	150	150	40	100	0	60	150	150
WS_D	140	150	30	200	0	100	100	100
	consumption							
	P1	**P2**						
WS_A	5	4						
WS_B	5	4						
WS_C	5	4						
WS_D	2	4						

Original planning problem: solution







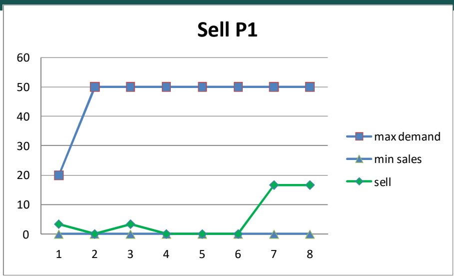


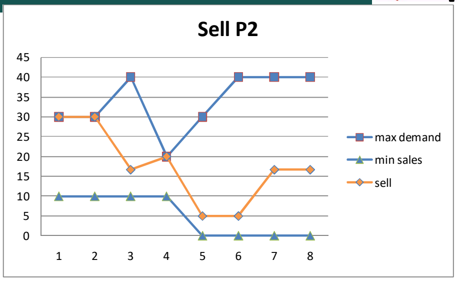
Original planning problem: modified example

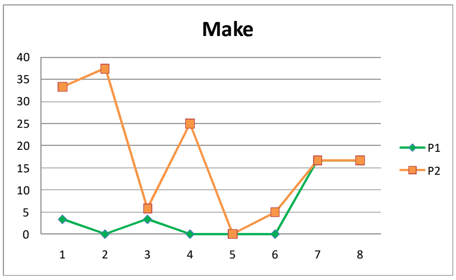
Products
- P2: higher profit, lower holding cost

period	1	2	3	4	5	6	7	8
product	**max demand**							
P1	20	50	50	50	50	50	50	50
P2	30	30	40	20	30	40	40	40
	min sales							
P1	0	0	0	0	0	0	0	0
P2	10	10	10	10	0	0	0	0
	profit		**holding cost**		**init hold**			
P1	1500		20		0			
P2	2000		5		0			

Solution of the modified example







Aggregated planning: material demand

Problem statement

- Similar to the original one, with additional constraints
- We need materials for production
  - Several products may need the same material
- Material demand is known for each product
- Total available material is limited
  - So-called non-renewable resource

Question

- How to maximize total profit by producing and selling products?
- Specifically: in each period and for each product
  - How much to produce?
  - How much to sell?
  - How much to stock?

Program

- aggregate_planning_material

Aggregated planning: material demand (2)

•New elements in the model

Index	
Materials	$k = 1,\dots,K$
Parameters	
Material need, per each product	b_{ik}
Available material quantities	B_k

•New constraints

- Total material usage is limited on the whole horizon

$$\sum_{t=1}^{T}\sum_{i=1}^{m} b_{ik} X_{it} \le B_k \quad \forall k$$

•No new optimization criterion

Aggregated planning: backorder

•Problem statement

- Similar to the original one, with additional constraints
- Non-satisfied demand will not be lost
  - There is the opportunity of backorder
- With extra cost

•Question

- How to maximize total profit by producing and selling of products?
- Specifically: in each period and for each product
  - How much to produce?
  - How much to sell?
  - How much to stock?
  - How much to backorder?

•Program

- Inventory position: Inventory minus backorder
  - May be negative
- aggregate_planning_backorder

Aggregated planning: backorder (2)

- New elements in the model

Parameters	
Backorder cost for each product, per time period	r_i
Decision variables	
Inventory position, in each period	I_{it}
Actual inventory, in each period	I_{it}^{+}
Backorder, in each period	I_{it}^{-}

- New optimization criterion
  - New profit maximum: also backorder decreases the profit

$$\max \sum_{t=1}^{T} \sum_{i=1}^{m} p_i S_{it} - h_i I_{it}^{+} - r_i I_{it}^{-}$$

Aggregated planning: backorder (3)

- New constraints
  - Inventory position

$$I_{it} = I_{it}^{+} - I_{it}^{-} \quad \forall i,t$$

  - Integrity constraints
    - Only the inventory position may be negative …

$$I_{it}^{+}, I_{it}^{-} \geq 0$$

- The balance equation remains the same

$$I_{it} = I_{it-1} + X_{it} - S_{it} \quad \forall i,t$$

Aggregated planning: backorder and overtime

- Problem statement
  - Similar to the original one, with additional constraints
  - Overtime is possible
    - Different resource by resource
  - Overtime takes extra cost
  - Depending on the various cost factors, compromise between backorder and overtime have to be found
- Question
  - How to maximize total profit by producing and selling products?
  - Specifically: in each period and for each product
    - How much to produce?
    - How much to sell?
    - How much to stock?
    - How much to backorder?
    - How much overtime is needed?
- Program
  - aggregate_planning_backorder_overtime

Aggregated planning: backorder and overtime (2)

New elements in the model

Parameters	
Overtime cost per time period, for each resource	q_j
Overtime limit, for each resource	L_j
Decision variable	
Overtime amount, for each machine and each period	O_{jt}

New optimization criterion

- Profit maximisation
  – But backorder and overtime decreases the profit

$$\max \sum_{t=1}^{T} \left(\sum_{i=1}^{m} p_i S_{it} - h_i I_{it}^{+} - r_i I_{it}^{-} - \sum_{j=1}^{n} q_j O_{jt} \right)$$

Aggregated planning: backorder and overtime (3)

New constraints

- Resource capacities extended with overtime

$$\sum_{i=1}^{m} a_{ij} X_{it} \leq c_{jt} + O_{jt} \quad \forall j,t$$

- Overtime limits $O_{jt} \leq L_j \quad \forall j,t$

- Integrity constraints $O_{jt} \geq 0$

Example: original problem

- Same demand and costs
  - But: very expensive backorder and overtime

period	1	2	3	4	5	6	7	8
product	max demand							
P1	20	50	50	50	50	50	50	50
P2	30	30	40	20	30	40	40	40
	min sales							
P1	0	0	0	0	0	0	0	0
P2	10	10	10	10	0	0	0	0
		cost			init			
	profit	holding	backorder		hold	backorder		
P1	1500	20	3000		0	0		
P2	1000	10	2000		0	0		

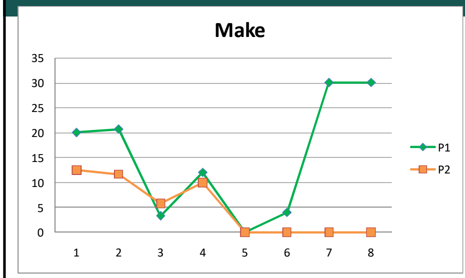
period	1	2	3	4	5	6	7	8
resource	capacity							
WS_A	250	250	60	100	0	20	200	240
WS_B	150	220	60	100	0	120	200	240
WS_C	150	150	40	100	0	60	150	150
WS_D	140	150	30	200	0	100	100	100
	consumption							
	P1	P2		overtime limit			overtime cost	
WS_A	5	4		100			1000	
WS_B	5	4		100			1000	
WS_C	5	4		100			1000	
WS_D	2	4		100			1000	

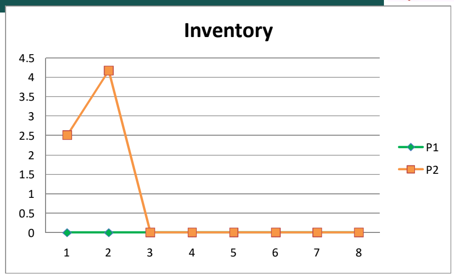
- Same solution
  - No backorder

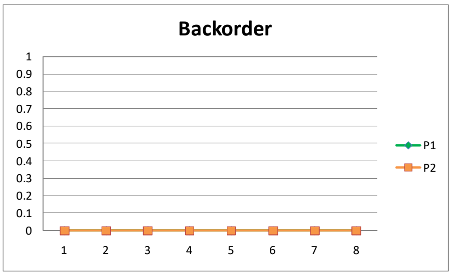
$$I_{it} = I_{it}^{+} \quad \forall i,t$$

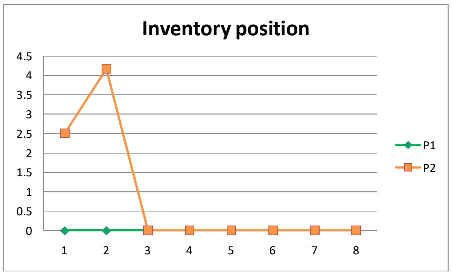
  - No overtime

Example: solution of the original problem









Example: modified problem

Less additional costs

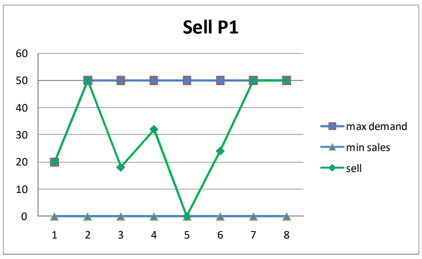
- Backorder, overtime

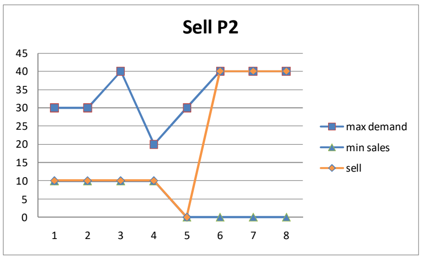
period	1	2	3	4	5	6	7	8
product	max demand							
P1	20	50	50	50	50	50	50	50
P2	30	30	40	20	30	40	40	40
	min sales							
P1	0	0	0	0	0	0	0	0
P2	10	10	10	10	0	0	0	0
		cost			init			
	profit	holding	backorder		hold	backorder		
P1	1500	20	500		0	0		
P2	1000	10	300		0	0		

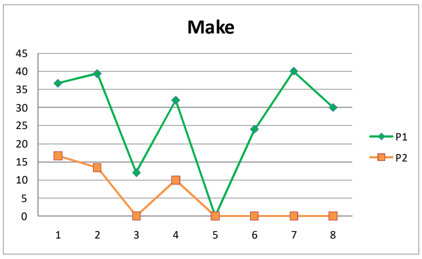
period	1	2	3	4	5	6	7	8
resource	capacity							
WS_A	250	250	60	100	0	20	200	240
WS_B	150	220	60	100	0	120	200	240
WS_C	150	150	40	100	0	60	150	150
WS_D	140	150	30	200	0	100	100	100
	consumption							
	P1	P2		overtime limit			overtime cost	
WS_A	5	4		100			100	
WS_B	5	4		100			100	
WS_C	5	4		100			100	
WS_D	2	4		100			100	

Example: solution of the modified problem

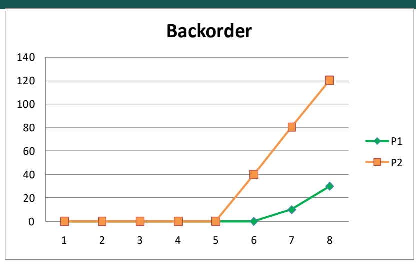
More production and sale quantities

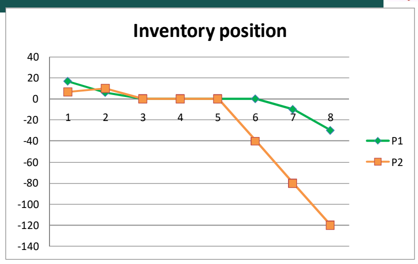


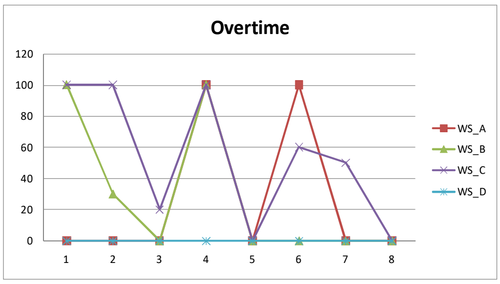




Example: solution of the original problem (2)







Aggregated planning: summary

- Dynamic models
- Linear programs
  - Available powerful solvers
- Incremental model extensions
  - With various kinds of constraints
  - Typical engineer's approach
- Efficient solution
  - What-if analysis
  - Sensitivity analysis

- But: not all problem types can be captured as an LP
  - E.g., deciding about the acquisition of a resource
  - Yes/No type decision (0/1 valued variable)
    - Search of the best solution
    - Much harder to find optima

Sources, contact

Textbook

- Hopp, W.J.; Spearman, M.L, *Factory physics, Foundations of manufacturing management*, Irwin, (second edition, 2000, third edition 2008).

Contaxt

- Dr. Váncza József
  - Phone: 279 6299
  - vancza@manuf.bme.hu

Thank You for your attention!

www.manuf.bme.hu